Mathematics of Engineering I

Brandenburgische Technische Universität Cottbus

Prof. Dr. R. Reemtsen, Th. Laue

Written Exam, February 9, 2010

Surname: .... **First Name:** ....

Student No.: ....

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**Σ**	**Grade**
Possible Points	12	4	10	7	8	9	50	
Obtained Points								

Books, tables, formularies, lecture notes, etc. can be used.

Calculators are not needed and not allowed.

The way by which a solution is found must be comprehensible.

x	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin(x)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos(x)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan(x)$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$		$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

Problem 1 *(4+2+3+3)*

a) For the complex numbers $z_1 = 2 + 5i$, $z_2 = 1 + 2i$ and $z_3 = -1 + i$ calculate

$$\mathrm{Re}(z_3), \qquad \mathrm{Im}(z_3), \qquad |z_3|, \qquad \text{and} \qquad z_4 = z_1 + 2\overline{z_3}z_2.$$

b) Find the cartesian form of

$$z = \frac{2i}{1-i}.$$

c) Represent the complex number

$$z = (1-i)^{10}$$

in polar form and in Euler form by using the principal argument for representing z.

d) Find all complex solutions of the equation

$$z^2 = \sqrt{8}(1-i).$$

Problem 2 *(4)*

Find all $x \in \mathbb{R}$ which solve the inequality

$$|x+2| < |5-x| + 2.$$

Problem 3 *(5+5)*
Consider the following system of linear equations depending on a parameter $\alpha \in \mathbb{R}$:

$$\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & + & 2x_3 & + & 2x_4 & = & 1, \\ & & x_2 & + & 2x_3 & - & x_4 & = & 1, \\ & - & x_2 & & & + & 2x_4 & = & -1, \\ & & x_2 & + & 4x_3 & + & (\alpha - 2)x_4 & = & 1. \end{array}$$

a) For which value(s) of α does the system have a unique solution? Evaluate this solution.

b) Determine the solution set of the system for $\alpha = 2$.

Problem 4 *(3+1+3)*
For the matrices

$$A = \begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} -1 & 4 & 0 \\ 3 & 0 & 2 \\ 0 & -2 & 1 \end{pmatrix}$$

determine

a) the sum $2A - 3C$ and the product AC,

b) the determinant of A,

c) the inverse matrix of A.

Problem 5 *(4+3+1)*
For the points $P_1 = (-1, 0, 1)$, $P_2 = (0, -1, 0)$, and $P_3 = (2, 0, 1)$ determine

a) the area of the triangle with vertices P_1, P_2, and P_3,

b) the Hesse normal form of the plane passing through P_1, P_2, and P_3,

c) the equation of the line which is perpendicular to the plane obtained in b) and which passes through the point $Q = (2, -1, 1)$.

Problem 6 *(5+4)*
Consider the matrix

$$A = \begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}.$$

a) Find all eigenvalues of A.

b) Calculate an eigenvector for the largest eigenvalue of A.